Design of Shear reinforcement in singly reinforced beam

Shear force: In case of beam, Shear force at any section is nothing but only the algebraic sum of all vertical forces of one side (either left or right side) from that section.
According to revised I.S. code

1. **Nominal shear stress** ($\tau_v$): it must be taken by beam safely.
   \[ \tau_v = \frac{s}{bd} \]
   Where, $s$= shear force due to design load. ($s = \frac{wL}{I}$)
   $b$= breadth of the section
   $d$ = effective depth of the section.

2. **Permissible nominal shear stress** ($\tau_c$): allowable shear stress which can be safely taken by beam. Its value can be determined by I.S. code 456 (page no 84, Table no 23).
   (value of $\tau_c$ are in N/mm$^2$, for intermediate % of steel use interpolation method)

<table>
<thead>
<tr>
<th>% STEEL = $\frac{A_m}{bd} \times 100$</th>
<th>M 15</th>
<th>M 20</th>
<th>M 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>≤ 0.15</td>
<td>0.18</td>
<td>0.18</td>
<td>0.19</td>
</tr>
<tr>
<td>0.25</td>
<td>0.22</td>
<td>0.22</td>
<td>0.23</td>
</tr>
<tr>
<td>0.50</td>
<td>0.29</td>
<td>0.30</td>
<td>0.31</td>
</tr>
<tr>
<td>0.75</td>
<td>0.34</td>
<td>0.35</td>
<td>0.36</td>
</tr>
<tr>
<td>1.00</td>
<td>0.37</td>
<td>0.39</td>
<td>0.40</td>
</tr>
<tr>
<td>1.25</td>
<td>0.40</td>
<td>0.42</td>
<td>0.44</td>
</tr>
<tr>
<td>1.50</td>
<td>0.42</td>
<td>0.45</td>
<td>0.46</td>
</tr>
</tbody>
</table>

3. **Maximum Nominal shear stress** ($\tau_{c,max}$): It is the maximum shear stress taken by any grade of concrete. It should never used by concrete for good sake. Its value can be found by using I.S. code 456 (page no 85, Table no 24).

<table>
<thead>
<tr>
<th>Concrete Grade</th>
<th>M 15</th>
<th>M 20</th>
<th>M 25</th>
<th>M 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{c,max}$ (N/mm$^2$)</td>
<td>1.6</td>
<td>1.8</td>
<td>1.9</td>
<td>2.2</td>
</tr>
</tbody>
</table>

4. **Minimum reinforcement in beam**:

   \[ \frac{A_{sv}}{bs_v} \geq \frac{0.4}{\sigma_y} \]

   Where,
   
   $A_{sv} =$ total c/s area of stirrup legs effective in shear, $(n \times \frac{\pi \theta^2}{4})$
   
   $s_v =$ spacing of stirrups along the length of the member centre to centre,
   
   $b =$ breadth of the beam,
   
   $\sigma_y =$ characteristics strength of the stirrup reinforcement in N/mm$^2$

**Case arises with $\tau_v$, $\tau_c$ & $\tau_{c,max}$**:
I. \( \tau_v < \tau_c < \tau_{c \text{ max}} \)

II. \( \tau_c < \tau_v < \tau_{c \text{ max}} \)

III. \( \tau_c < \tau_{c \text{ max}} < \tau_{v} \)

**CASE 1:** \( \tau_v < \tau_c < \tau_{c \text{ max}} \)

Since in this case value of nominal shear stress (\( \tau_v \)) is less than the allowable shear stress (\( \tau_c \)). so beam is safe in shear and there is no need of shear reinforcements.

But according to I.S. 456, Minimum Shear Reinforcement should be provided.

\[
\frac{A_{sv}}{b s_v} \geq \frac{0.4}{\sigma_y}
\]

**CASE 2:** \( \tau_c < \tau_v < \tau_{c \text{ max}} \)

Since in this case value of nominal shear stress (\( \tau_v \)) is greater than the allowable shear stress(\( \tau_c \)). so beam is not safe in shear and there is need of shear reinforcements.

There are two types of sub-cases arise:

**Sub-case 1: No bar has been cranked.**

\[
s_v \leq \frac{A_{sv} \times \sigma_{sv} \times d}{V_s}
\]

Where,

\( s_v = \) Spacing of stirrups along the length of beam centre to centre.

\( V_s = (\tau_v - \tau_c) \times b \times d \)

\( d = \) effective depth of the beam

\( \sigma_{sv} = \) permissible stress in stirrup = \( \sigma_{st} \)

\( A_{sv} = \) total c/s area of stirrup legs effective in shear, \( (n \times \frac{\pi d^2}{4}) \)
Sub-case 2: Some alternate longitudinal bars have been cranked.

For cranked bar section:

\[
S_v \leq \frac{A_{sv} \times \sigma_{sv} \times d}{V_i}
\]

\[
V_i = V_s - \text{No. of bars cranked} \times \sigma_{sv} \times \frac{\pi \varnothing^2}{4} \times \sin \alpha
\]

\[
\Rightarrow V_i = V_s - f
\]

\( f \) should be less than \( \frac{V_s}{2} \)

If \( f > \frac{V_s}{2} \), take it equal to \( \frac{V_s}{2} \).
**CASE 3:** $\tau_c < \tau_{c,\text{max}} < \tau_v$

This type of case is result of wrong design. We have to redesign the beam by changing $b$ and $d$ or changing in concrete & steel grade if possible.

5. **Minimum Spacing of stirrups centre to centre:**

The spacing of stirrups shall not exceed the following:

- **I.** $300$ mm
- **II.** $0.75 \, d$
- **III.** $s_v \leq \frac{0.87 \, \sigma_{ck}}{0.4} \frac{A_{st}}{b}$

**Example 1:** An R.C. beam of span $5$ m is $250$ mm wide and $500$ mm deep to the centre of tensile reinforcement, which consist of $4$ bars of $22$ mm diameter. The beam carries a load of $30$ KN/m inclusive of its weight. Design the shear reinforcement by stirrups. Use $M_{20}$ concrete and $Fe_{415}$ steel.

**Sol:** Given data:

- $l = 5 \, m$
- $b = 250 \, mm$
- $d = 500 \, mm$

$A_{st} = n \times \frac{\pi}{4} b^2 = 4 \times \frac{\pi}{4} 22^2 = 1520 \, mm^2$

$w = 30 \, KN/m$

Max $^m$ shear force in SSB, $s = \frac{wl}{2} = \frac{30 \times 5}{2} = 75 \, KN$

Nominal Shear Stress, $\tau_v = \frac{s}{bd} = \frac{75 \times 10^3}{250 \times 500} = 0.60 \, N/mm^2$

Percentage of steel provided $= \frac{A_{st}}{bd} \times 100 = \frac{1520}{250 \times 500} \times 100 = 1.22\%$

From I.S. 456, (page no 84, Table no 23).

For $M_{20}$ grade of concrete and $1.22\%$ steel,

Since for $1.22\%$ steel value of permissible stress ($\tau_c$) is not given so we will find out by using Interpolation formula.

<table>
<thead>
<tr>
<th>% of steel</th>
<th>Permissible shear stress ($\tau_c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.39</td>
</tr>
<tr>
<td>1.25</td>
<td>0.42</td>
</tr>
</tbody>
</table>
By using interpolation formula:

$$\tau_c = 0.39 + \frac{(1.22-1.00)}{(1.25-1.00)} \times (0.42 - 0.39) = 0.416 \text{ N/mm}^2$$

Now from I.S. code 456 (page no 85, Table no 24),

$$\tau_{c\text{ max}} = 1.8 \text{ N/mm}^2$$, for $M_{20}$ concrete

Now comparing values, it is clear that;

$$\tau_c < \tau_U < \tau_{c\text{ max}}$$, Hence need to provide shear reinforced.

Let us provide 2 legged 8 mm Ø stirrups.

\[ A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100 \text{ mm}^2 \]

Since there is no any bar cranked so spacing of stirrups c/c;

\[ s_v \leq \frac{A_{sv} \times \sigma_{sv} \times d}{V_s} = \frac{100 \times 230 \times 500}{(0.60 - 0.416) \times 250 \times 500} = 500 \text{ mm} \]

Spacing of stirrups shall not exceed the following:

1. 300 mm
2. \[ 0.75 \times d = 0.75 \times 500 = 375 \text{ mm} \]
3. \[ s_v \leq \frac{0.87 \sigma_{ck}}{0.4} \frac{A_{sv}}{b} = 361 \text{ mm} \]

So, provide 2 legged stirrups @ 300 mm c/c.
Example 2: An R.C. beam of span 6.50 m is 300 mm wide and 750 mm deep to the centre of tensile reinforcement which consist of 6 bars of 20 mm diameters. The beam carries a load of 45 KN/m including its weight. Design the shear reinforcement if 50 % of the tensile reinforcement is curtailed near the support. Use M<sub>20</sub> concrete and Fe<sub>415</sub> steel.

Sol:

Given data:

\[ l = 6.50 \text{ m} \]
\[ b = 300 \text{ mm} \]
\[ d = 750 \text{ mm} \]
\[ A_{st} = 6 \times \frac{\pi}{4} \times 20^2 = 1884.955 \text{ mm}^2 \]
\[ W = 45 \text{ KN/m} \]

50 % tensile reinforcement is curtailed.

So area of steel at supports, \( A'_{st} = 50 \% \text{ of } A_{st} = 942 \text{ mm}^2 \)

Max shear force in SSB, \( S = \frac{Wl}{2} = \frac{45 \times 6.50}{2} = 146.25 \text{ KN} \)

Nominal Shear Stress, \( \tau_v = \frac{S}{bd} = \frac{146.25 \times 10^3}{300 \times 750} = 0.65 \text{ N/mm}^2 \)

Percentage of steel at support \( = \frac{A'_{st}}{bd} \times 100 = \frac{942}{300 \times 750} \times 100 = 0.42\% \)

From I.S. 456, (page no 84, Table no 23),

For M<sub>20</sub> grade of concrete and 0.42 % steel,

Since for 0.42% steel value of permissible stress \( (\tau_c) \) is not given so we will find out by using Interpolation formula.

<table>
<thead>
<tr>
<th>% of steel</th>
<th>Permissible shear stress ( (\tau_c) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.22</td>
</tr>
<tr>
<td>0.50</td>
<td>0.30</td>
</tr>
</tbody>
</table>

By using interpolation formula:

\[ \tau_c = 0.22 + \frac{(0.42 - 0.25)}{(0.50 - 0.25)} \times (0.30 - 0.22) = 0.27 \text{ N/mm}^2 \]

Now from I.S. code 456 (page no 85, Table no 24),

\[ \tau_{c \text{ max}} = 1.8 \text{ N/mm}^2, \text{ for M}_{20} \text{ concrete} \]

Now comparing values, it is clear that;

\[ \tau_c < \tau_v < \tau_{c \text{ max}}, \text{ Hence need to provide shear reinforced.} \]
Let us provide 2 legged 8 mm Ø stirrups.

\[ A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100 \text{ mm}^2 \]

Spacing of stirrups c/c;

\[ s_v \leq \frac{A_{sv} \times \sigma_{sv} \times d}{V_s} = \frac{100 \times 230 \times 750}{(0.65 - 0.27) \times 300 \times 750} = 201.75 \text{ mm} \]

Spacing of stirrups shall not exceed the following:

1. 300 mm
2. 0.75 \( d \) = 0.75 \times 750 = 562.5 mm
3. \[ s_v \leq \frac{0.87 \sigma_{ck}}{0.4} \times \frac{A_{sv}}{d} = 300.875 \text{ mm} \]

So, provide 2 legged stirrups @ 300 mm c/c.